

Computing with M_3 : Multi-qubit interference

Deutsch–Jozsa and Bernstein–Vazirani. Existing algorithm slides are an outline, not a completed lesson.

THE CENTRAL QUESTION

How does the one-bit idea scale to an n-bit constant-or-balanced promise?

Hadamards turn the oracle's phase pattern into a deterministic all-zero test for constant functions.

Prediction: state what should change in the first experiment and what must remain invariant.

KEY TERMS

n-bit promise
phase oracle
Walsh-Hadamard transform
all-zero outcome

MODEL

amplitude of $|0\dots 0\rangle = \text{average}_x (-1)^{f(x)}$

Deutsch-Jozsa Algorithm

Given a function

$$f : \{0, 1\}^n \rightarrow \{0, 1\}$$

guaranteed to be either:

- **Constant:** the output is the same for all 2^n inputs, or
- **Balanced:** the output is 0 for exactly half of the inputs, and 1 for the other half,

determine which case applies using as few evaluations of f as possible.

In other words, we generalize to the case where the problem size is n instead of 2.

Deutsch-Jozsa Algorithm

In the worst case, a classical algorithm needs to evaluate up to: $2^{n-1} + 1$ times to be sure—because you could see 2^{n-1} zeros before hitting a one.

The Deutsch-Jozsa algorithm requires exactly 1 call to the quantum oracle. All 2^n inputs are explored in superposition, and quantum interference reveals the answer after a single evaluation.

Input Size n	Classical (Worst Case)	Quantum
1	2	1
2	3	1
3	5	1
n	$2^{n-1} + 1$	1
100	$\approx 6.3 \times 10^{29}$	1

THE CENTRAL QUESTION

How do we know a circuit really implements the Boolean oracle we intended?

A quantum algorithm is only as correct as its reversible oracle and bit-order contract.

KEY TERMS

reversible synthesis
ancillas
bit order
semantic tests

MODEL

classical truth table == basis-state action of U_f

Prediction: state what should change in the first experiment and what must remain invariant.

Deutsch–Jozsa derivation

Specify the constant-or-balanced promise and reversible oracle.

Derive the all-zero amplitude, then trace one constant and one balanced two-bit example.
Compare exact and randomized classical query baselines.

Bernstein–Vazirani derivation

Define the hidden bit string and parity oracle.

Show how phase kickback and the final Hadamards reveal the string. Include a hand-worked example and its classical query comparison.

Oracle cost and model limits

State what counts as a query. Separate query complexity from the cost of constructing the oracle.

Include a check that ancillas return to their intended state.