

Information in physical quantum systems

Existing teaching outlines plus clearly marked material for Bruna to develop.

Reversible Laws, Emergent Arrow of Time

Rules of our universe...

Fundamental Reversible Dynamics

Quantum (Schrödinger)

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

Fields (Maxwell)

$$\begin{aligned}\nabla \cdot E &= \rho_{\text{free}} & \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} & \nabla \times B &= \frac{\mu_0}{c^2} \frac{\partial E}{\partial t} + \mu_0 J\end{aligned}$$

Spacetime (Einstein)

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The Arrow of Time

Entropy Increases



Melting Ice



Broken Egg



Spreading Smoke

Order → Entropy → Higher Disorder

THE CENTRAL QUESTION

How many bits are required, on average, to describe an uncertain source?

Shannon entropy measures expected information and sets the asymptotic compression limit for a source.

Prediction: state what should change in the first experiment and what must remain invariant.

KEY TERMS

self-information
entropy
joint distribution
mutual information

MODEL

$$H(X) = -\sum_x p(x) \log_2 p(x)$$

FIRST WORKED EXPERIMENT

Start with one case small enough to check by hand.

1. Write the input and prediction
2. Apply the operation
3. Compare the observed evidence

Evidence to keep: normalised distribution, entropy curve, and coding overhead. Label basis order, shots, and simulator assumptions.

EXPERIMENT

Vary a binary source bias and compare entropy with empirical code length.

THE CENTRAL QUESTION

How should uncertainty be measured when a state has coherence and mixedness?

Von Neumann entropy is the Shannon entropy of a density matrix's eigenvalue distribution.

KEY TERMS

density matrix

eigenvalues

pure/mixed state

reduced state

MODEL

$$S(\rho) = -\text{Tr}(\rho \log_2 \rho)$$

Prediction: state what should change in the first experiment and what must remain invariant.

FIRST WORKED EXPERIMENT

Start with one case small enough to check by hand.

1. Write the input and prediction
2. Apply the operation
3. Compare the observed evidence

Evidence to keep: trace, positivity, purity, eigenvalues, and entropy. Label basis order, shots, and simulator assumptions.

EXPERIMENT

Compare pure, classically mixed, and reduced Bell-state density matrices.

Density matrices and reduced states

Develop pure states, statistical mixtures, and the partial trace with worked two-level examples.

Show how a pure entangled joint state gives a mixed local state.

Entropy and physical information

Calculate Shannon and von Neumann entropy in small examples.

Connect the eigenvalues of a density matrix to uncertainty, purity, and correlations. Define all conventions.