

Computing with M_3 : Amplitude amplification

Grover search as a pair of reflections. The existing material is an outline requiring worked examples.

THE CENTRAL QUESTION

How can two reflections rotate amplitude toward a marked state?

The oracle and diffuser act as reflections whose product amplifies the marked subspace.

Prediction: state what should change in the first experiment and what must remain invariant.

KEY TERMS

phase oracle

diffuser

two-dimensional rotation

marked state

MODEL

```
theta = asin(sqrt(M/N)); success =  
sin^2((2r+1)theta)
```

THE CENTRAL QUESTION

When does amplitude amplification help, and when does another iteration hurt?

The useful iteration count depends on the marked fraction; over-rotation is predictable.

KEY TERMS

multiple solutions
optimal iteration
over-rotation
noise accumulation

MODEL

r_{opt} is approximately $\pi/(4 \theta) - 1/2$

Prediction: state what should change in the first experiment and what must remain invariant.

A complete four-item search

Choose one marked state and define the phase oracle.

Write all four amplitudes before and after the oracle and diffuser. Derive the success probability without software.

Grover rotation and stopping

Derive the two-dimensional rotation and an appropriate iteration count.

Show over-rotation, multiple marked items, and the assumptions behind the quadratic query advantage.

Search resources and limits

Separate oracle calls, oracle construction, state preparation, and measurement cost.

Replace the ambiguous average-step and hardware-size claims in original Lecture 10, slides 8–9.