

Observables and measurement

Hermitian observables and physical measurement. The Qiskit measurement example moves to the final Q3 meeting.

FROM MEASUREMENT TO OBSERVABLES

What we know so far:

- A quantum state is a complex vector $|\psi\rangle$
- Measurement gives a real probabilistic outcome
- Probabilities come from amplitudes (Born rule)
- Measurement acts like a projection onto a basis

MEASUREMENTS LEAD TO OBSERVATIONS

Measurement

The *process* or *operation* you perform

- An interaction with a system
- Defined by:
 - what you measure (observable)
 - how you measure it (apparatus / basis)

Observation (or outcome)

The *result* you obtain from that measurement

- A specific value
- One of the possible outcomes of the measurement

E.g. Measurement: use a ruler

Observation: “the length is 12.3 cm”

The state of the system is independent of measurement.

E.g. Measurement: measure spin along the z-axis

Observation: “spin up” ($|0\rangle$) or “spin down” ($|1\rangle$)

In quantum mechanics, the measurement **affects the system**, not just reveals it.

The state of the system has an undecided state until measures.

When we measure a quantum system it *collapses* the undecided state into a particular observable following Born’s rule.

You will hear the term *collapse* a lot.

A Historical Note

In the 1920s, two competing views of quantum mechanics emerged:

Matrix mechanics (Heisenberg)

- Focused on operators and eigenvalues
- Abstract, algebraic

Wave mechanics (Schrödinger)

- Focused on wave equations
- Continuous, geometric

Erwin Schrödinger:

“Now the damned Göttingeners are using my beautiful wave mechanics to calculate their damn matrix elements.”

What we now understand

These are two descriptions of the same structure



Erwin Schrödinger

University of Vienna

“Die gegenwärtige Situation in der Quantenmechanik” (1935)
(*“The Present Situation in Quantum Mechanics”*)

SHRODINGERS WAVE EQUATION (AN ASIDE)

Schrödinger's Wave Equation (Preview)

The evolution of a quantum state is governed by:

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x), \psi(x, t)$$

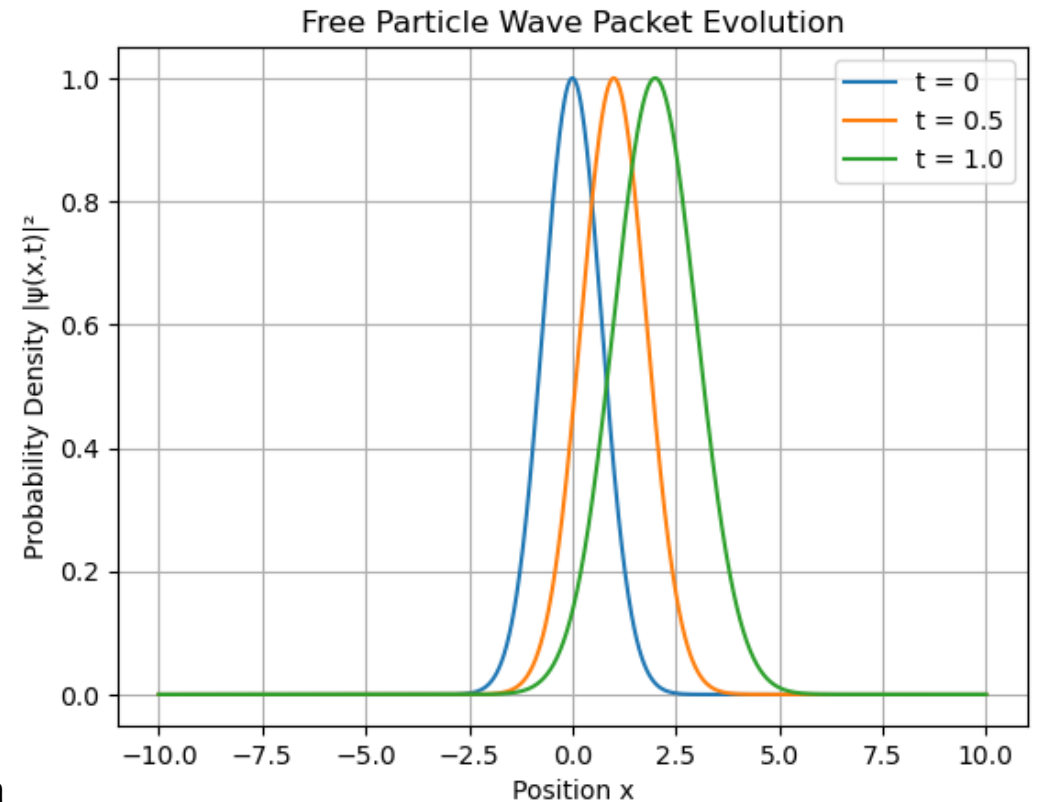
What this means (briefly)

$\psi(x, t)$: the quantum state (wavefunction)

Left side: how the state changes in time

Right side: how curvature and potential shape the evolution

This equation tells us how quantum states evolve when
are not measuring them



SCHRÖDINGER'S* CAT

- A cat in a sealed box
- A quantum event (radioactive decay)
- If decay occurs → poison released → cat dies
- If not → cat remains alive

Quantum mechanics:

$$|\psi\rangle = \alpha|\text{alive}\rangle + \beta|\text{dead}\rangle$$

So before observation, the cat is described as a **superposition of alive and dead**.

Once we measure the system collapses to a concrete measurement: alive or dead.



SCHRÖDINGER'S* CAT

- A cat
- A qu
- If de
- If no

As often happens Schrodinger created this thought experiment in an attempt to show how ridiculous Born's view was.

The “Big Bang” theory was another sarcastic description by Fred Hoyle that became adopted by the proponents.

Quant

So bef
super

Einstein's “spooky action at a distance” was meant as a complaint, but now appears in textbooks as a perfectly respectable phenomenon.

Once
concre

Physics has a habit of keeping the ideas and repurposing the insults.

TWO EQUIVALENT DESCRIPTIONS OF QUANTUM MECHANICS

Schrödinger

State: a function $\psi(x, t)$

- Evolution: differential equation
- waves, continuous space

Heisenberg / Dirac

- State: a vector $|\psi\rangle$
- Evolution: matrix (operator) multiplication
- linear algebra

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Where wave mechanics is still used

- Quantum chemistry (solving Schrödinger equation numerically)
 - Physics simulations (continuous systems)
 - Some analog/optical quantum systems
-

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In this lecture:

What determines *which* outcomes are possible?

Where do the measurement values come from?

Why are some directions in which we are allowed to project special?

OBSERVABLES ARE OPERATORS

An observable is defined by an operator (linear map)

- **This operator determines:**
- **Special directions** $\rightarrow$ eigenvectors
- **Observed values** $\rightarrow$ eigenvalues

When we measure a state $|\psi\rangle$:

- It is projected onto one of the operator's eigenvectors
 - The outcome is the corresponding eigenvalue
-

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OBSERVABLES ARE OPERATORS

- There are many examples of observing quantum systems. (e.g 2-slit experiment, here is another)

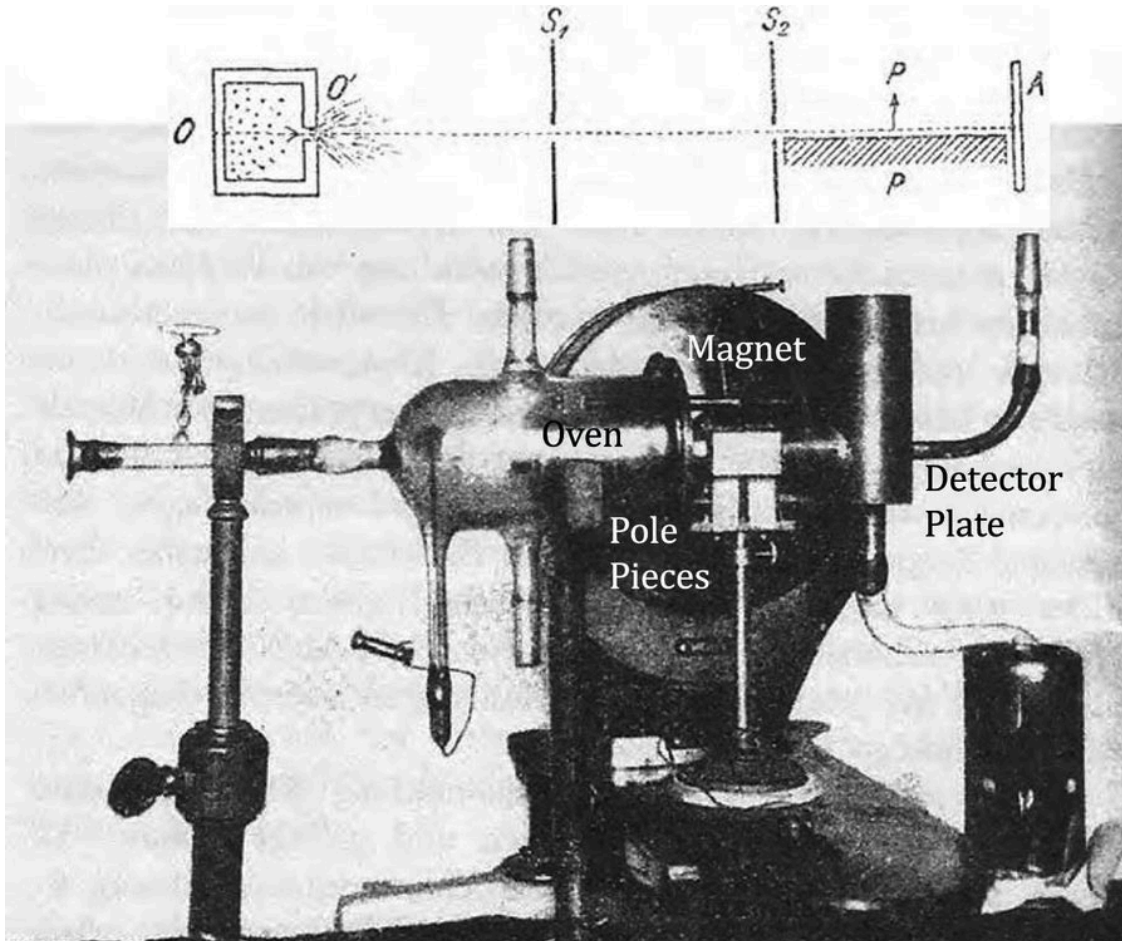
Example: Electron Spin Measurement (Stern–Gerlach)

- Observable: **spin along an axis** (e.g. Z)
- Outcomes: $+\frac{1}{2}$ or $-\frac{1}{2}$ (or “up / down”)

Shoots silver atoms through a magnetic field gradient

Beam splits into two discrete paths
The two paths correspond to **eigenvectors**: $|0\rangle$ and $|1\rangle$ with respect to the Z axis*.

The two outcomes correspond to **eigenvalues**: $\frac{1}{2}$ and $-\frac{1}{2}$



(A). Credit: “Otto Stern’s Molecular Beam Method and Its Impact on Quantum Physics,” by Bretislav Friedrich and Horst Schmidt-Böcking, in *Molecular Beams in Physics and Chemistry*. Edited by Bretislav Friedrich and Horst Schmidt-Böcking. Springer, 2021(CC BY 4.0)

* $|\uparrow\rangle$ means “spin up along Z”
 $|\downarrow\rangle$ means “spin down along Z”

A CONCRETE OBSERVABLE (Z MEASUREMENT)

Measure along the Z direction

We already know its directions:

$|0\rangle \rightarrow$ north pole

$|1\rangle \rightarrow$ south pole

Define the operator

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

What this means

$$Z|1\rangle = -1 \cdot |1\rangle \quad Z|0\rangle = +1 \cdot |0\rangle$$

Interpretation

$|0\rangle \rightarrow$ outcome +1

$|1\rangle \rightarrow$ outcome -1

A CONCRETE OBSERVABLE (X MEASUREMENT)

Measure along the X direction

We already know its directions:

$|+\rangle \rightarrow +X$ direction

$|-\rangle \rightarrow -X$ direction

Define the operator

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

What this operator does

$$X|1\rangle = |0\rangle X|0\rangle = |1\rangle$$

In the Z basis this mixes states together

In the X basis it moves us up and down the possible observed values.

$$X|-\rangle = -1 \cdot |-\rangle X|+\rangle = +1 \cdot |+\rangle$$

Interpretation

$|+\rangle \rightarrow$ outcome **+1**

$|-\rangle \rightarrow$ outcome **-1**

MEASUREMENT

Measurement	Directions	Outcomes
Z	$ 0\rangle, 1\rangle$	± 1
X	$ +\rangle, -\rangle$	± 1

Measurement mismatch:

Start with a known state:

$$|\psi\rangle = |0\rangle$$

Measure in the Z basis

Outcome: always $|0\rangle$

Value: +1

MEASUREMENT IN THE CORRECT BASIS

Now measure in the X basis

First rewrite the state:

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

Apply the Born rule

$$P(-) = \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2} \quad P(+) = \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2}$$

Result

- 50% $\rightarrow |+\rangle$ (value: +1)
- 50% $\rightarrow |-\rangle$ (value: -1)

The system didn't change. By asking a different question (as defined by X and Z) we got a different set of answers.

The measurement is defined by our choice of observable.

RECAP

A measurement is defined by an operator A

The operator determines:

- **Special directions** $\rightarrow$ the only possible outcomes
- **Values attached to those directions** $\rightarrow$ what we observe

When we measure a state:

- The state is projected onto one of these directions
- We observe the value associated with that direction

Linear algebra language

These special directions are called **eigenvectors**

The observed values are **eigenvalues**

Summary

Measurement = projection onto eigenvectors

Observation = corresponding eigenvalue

Stern–Gerlach measurement model

Develop the preparation, magnetic-field gradient, and readout.

Separate the Pauli observable with eigenvalues ± 1 from physical spin angular momentum with eigenvalues $\pm \hbar/2$.

Sequential measurements

Work through two measurement axes and conditional probabilities.

Explain the change of state and the difference between an observable, apparatus setting, and observed value.