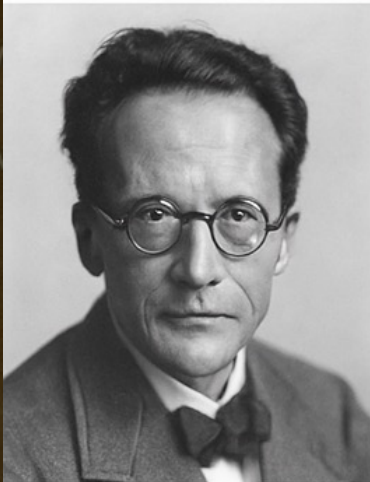


Waves, amplitudes, and interference

Physical evidence and amplitude addition. Gate synthesis and algorithms belong in Q3.

The Quantum Revolution

Foundations of Quantum Science



Erwin Schrödinger

1887–1961

Austrian

University of Vienna

$$i\hbar\partial\Psi = H\Psi$$



Max Born

1882–1970

German

University of Göttingen

$$|\psi|^2$$



Werner Heisenberg

1901–1976

German

University of Leipzig

$$\Delta x \Delta p \geq \frac{h}{2}$$



Louis de Broglie

1892–1987

French

Collège de France

$$\lambda = \frac{h}{p}$$

Wave-Particle Duality

Demonstration of single-electron buildup of an interference pattern

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The wave-particle duality of electrons was demonstrated in a kind of two-slit interference experiment using an electron microscope equipped with an electron biprism and a position-sensitive electron-counting system. Such an experiment has been regarded as a pure thought experiment that can never be realized. This article reports an experiment that successfully recorded the actual buildup process of the interference pattern with a series of incoming single electrons in the form of a movie.

I. INTRODUCTION

The two-slit interference experiment with electrons is frequently discussed in textbooks on quantum mechanics, and is referred to as “impossible, absolutely impossible to explain in any classical way, and has in it the heart of quantum mechanics.”¹ In this experiment (see Fig. 1), electrons incident on a wall with two slits pass through the slits and are detected one by one on a screen behind them. Accumulation of successive single electrons detected at the screen builds up an interference pattern. According to the interpretation in quantum mechanics, a single electron can pass through both of the slits in a wave form called “probability amplitude” when the uncertainty of the electron position in the wall plane covers the two slits, and when no observation is made of the electron at either one of the slits. The electron is then detected as a particle at a point somewhere on the screen according to the probability distribution of the interference pattern. However, if the electron is caught when passing through the slits, it takes place at either one of the two slits, never both, and the probability distribution on the screen will be completely different.

Although in textbooks this experiment is talked about as

a matter of fact, “this experiment has never been done in just this way, since the apparatus would have to be made on an impossibly small scale,” as Feynman points out.¹ However, this is not necessarily true. In fact, several attempts have been made up to now; Zeilinger *et al.*² confirmed the

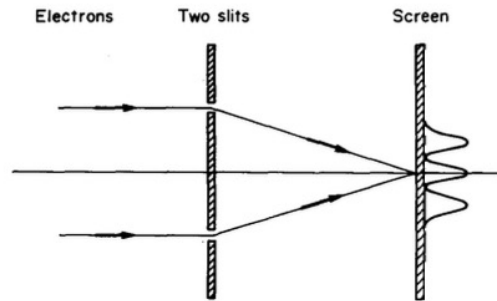


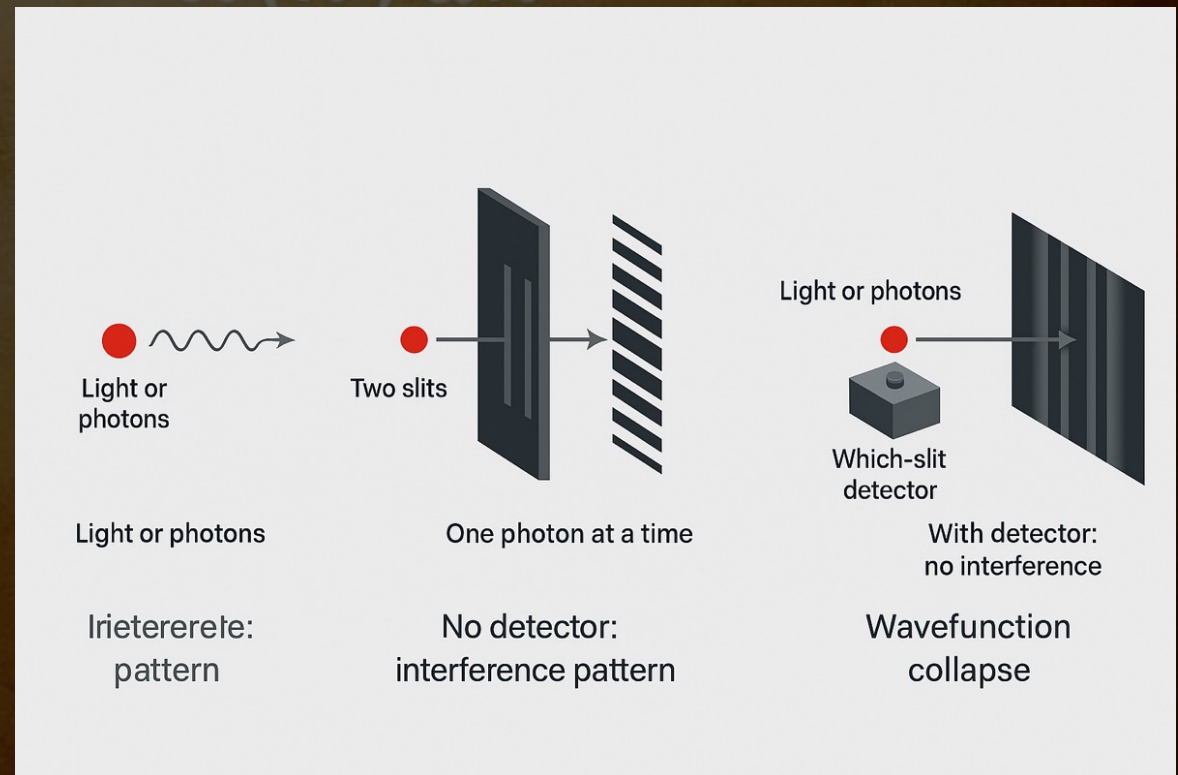
Fig. 1. Two-slit electron interference experiment.

Einstein (1905): Light quanta (photons) to explain the photoelectric effect

• **de Broglie (1924):** Hypothesised that matter (like electrons) also has wave properties

Davisson–Germer (1927): Confirmed electron diffraction, validating de Broglie

Bohr, Heisenberg, Schrödinger (1920s): Formalised the quantum framework



Wave-Particle Duality

Einstein (1905): Light quanta (photons) to explain the photoelectric effect
•de Broglie (1924): Hypothesised that matter (like electrons)

Demonstration of single electron diffraction,

A. Tonomura, J. Endo
Advanced Research Laboratory
H. Ezawa
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(Received 17 December 1989)

The wave-particle duality of matter is demonstrated in a single electron diffraction experiment using an extremely sensitive electron-counting system. The experiment is a variation of the Davisson-Germer experiment that can not only record the actual build-up of the diffraction pattern but also record the actual build-up of electrons in the form of

I. INTRODUCTION

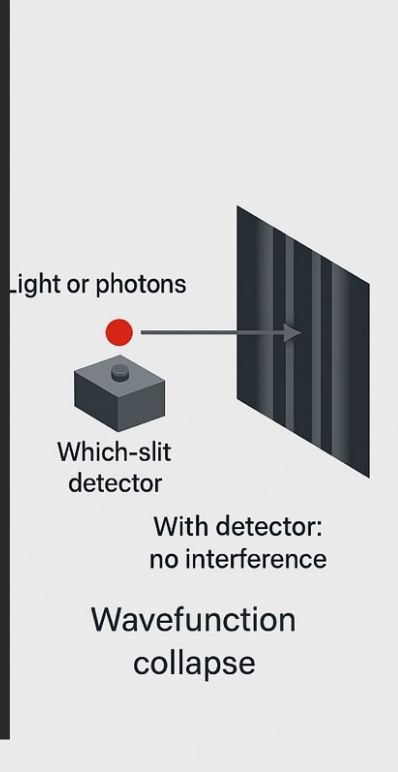
The two-slit interference experiment is frequently discussed in textbooks and is referred to as “impossible to explain in any classical way, and only quantum mechanics.”¹ In this experiment, electrons incident on a wall with two slits and are detected one by one on a screen. As a result, a simulation of successive single electrons passing through the slits builds up an interference pattern. The interpretation in quantum mechanics is that the electron passes through both of the slits in a “probability amplitude” when the uncertainty in the wall plane covers the slits. When an observation is made of the electron, the electron is then detected as a particle at a specific location on the screen according to the probability distribution of the interference pattern. If the electron is caught when passing through either one of the two slits, never mind the probability distribution on the screen will be a single-slit distribution.

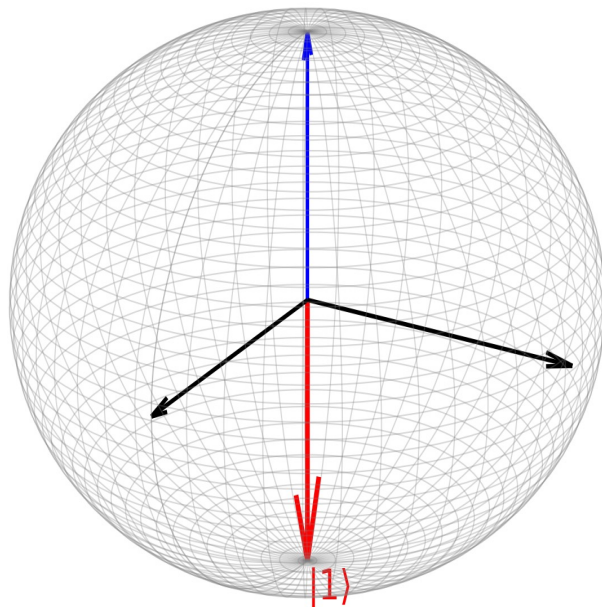
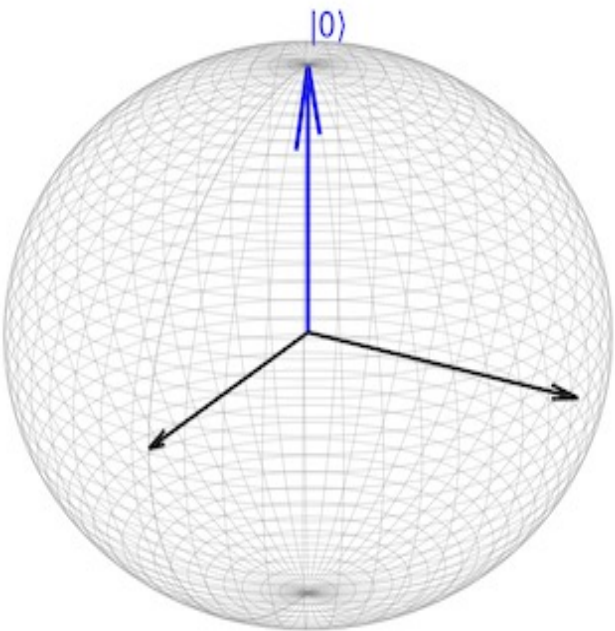
Although in textbooks this experiment is usually described as a demonstration of the wave-particle duality of matter, it is actually a demonstration of the wave-particle duality of matter.



electron diffraction,

): Formalised the

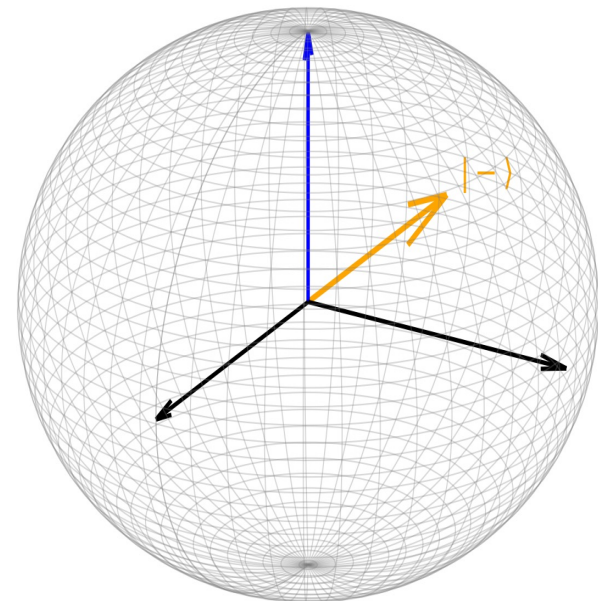
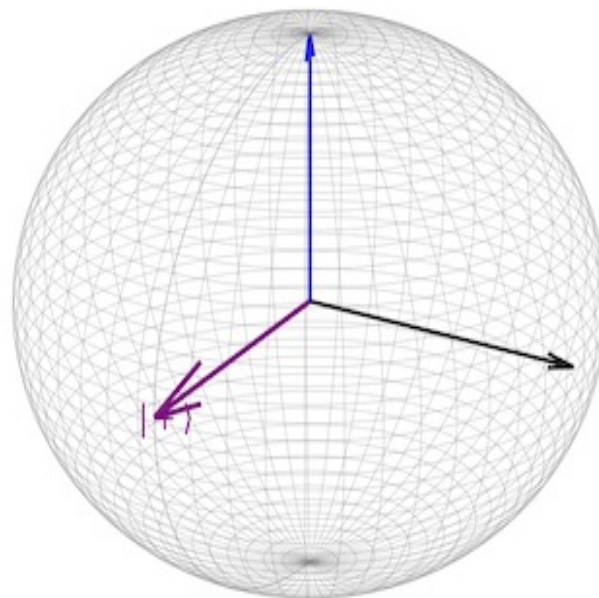




$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Standard Basis



$$|+\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad |-\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Hadamard Basis

Quantum Interference

$$P(x) = |\psi(x)|^2$$

Concept

Like in waves

Constructive

Destructive

Why it matters

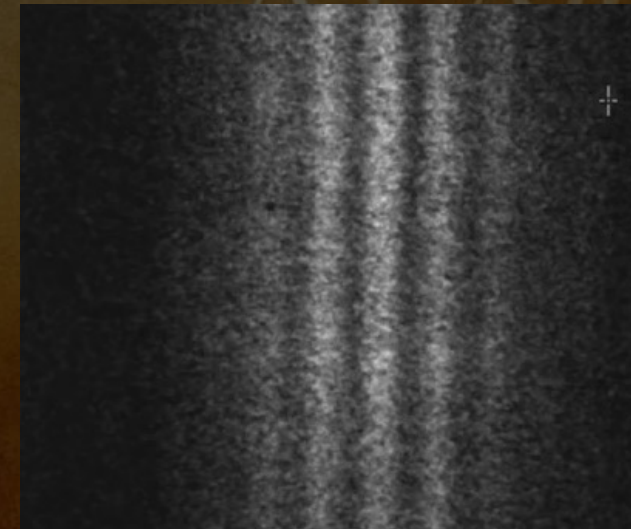
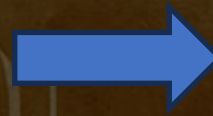
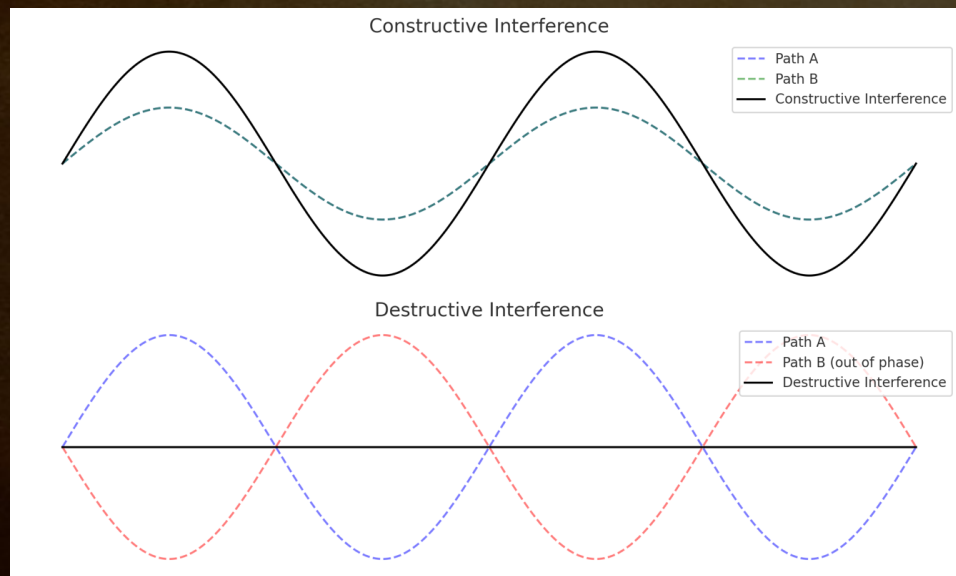
Quantum Interference

Quantum amplitudes can add or cancel

Amplitudes **reinforce**, increasing outcome probability

Amplitudes **cancel**, suppressing outcomes

Used in quantum algorithms to find correct answers faster



USING CLASSICAL RULES TO FIND THE PROBABILITY OF LANDING AT POSITION X

Classical Rule: Probabilities Add

Suppose a particle can arrive at screen location x in two mutually exclusive ways:

- through slit 1: probability $P_1(x)$
- through slit 2: probability $P_2(x)$

Then the total probability is: $P(x) = P_1(x) + P_2(x)$

- $P_1(x)$: chance of arriving at x through slit 1
- $P_2(x)$: chance of arriving at x through slit 2

So:

total probability = sum of the two path probabilities

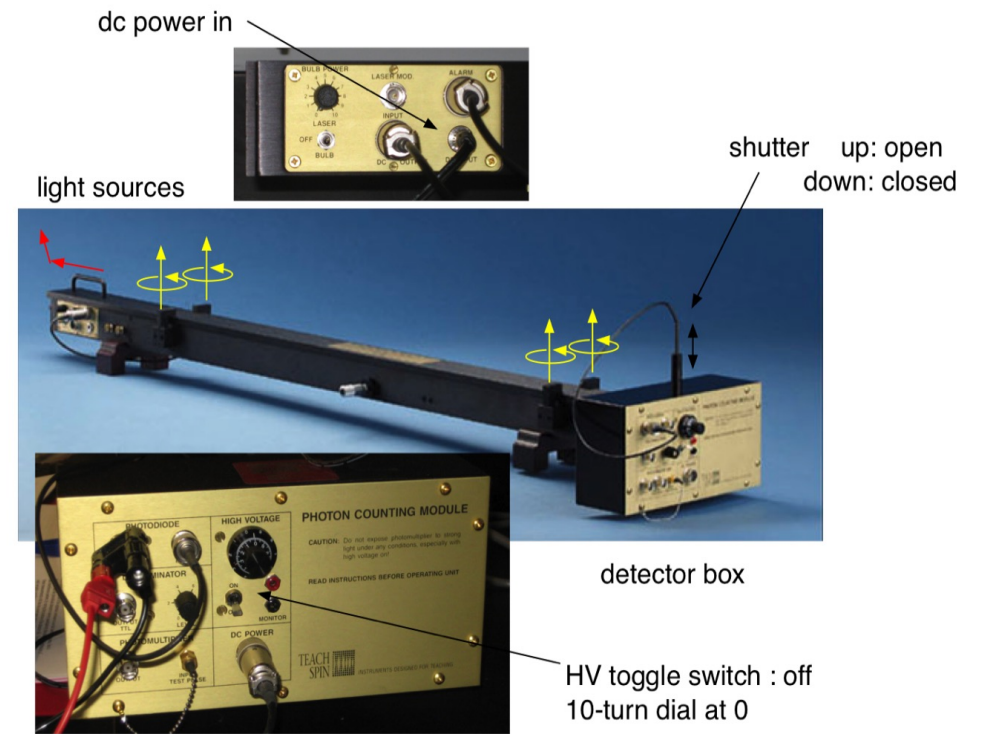


Fig. 15.1 Overview of the two-slit interference apparatus.

USING CLASSICAL RULES TO FIND THE PROBABILITY OF LANDING AT POSITION X

Classical Rule: Probabilities Add

Suppose a particle can arrive at screen location x in two mutually exclusive ways:

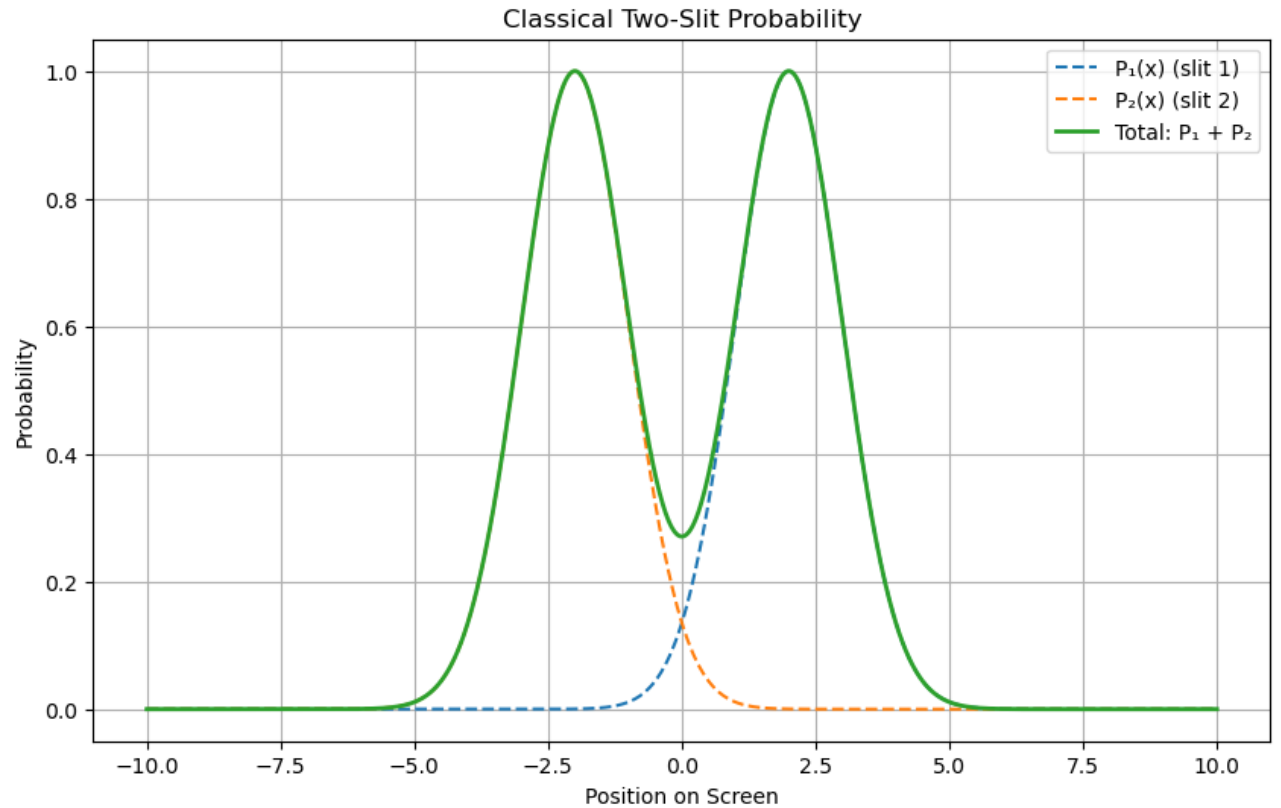
- through slit 1 $\rightarrow$ probability $P_1(x)$
- through slit 2 $\rightarrow$ probability $P_2(x)$

Then the total probability is: $P(x) = P_1(x) + P_2(x)$

- $P_1(x)$: chance of arriving at x through slit 1
- $P_2(x)$: chance of arriving at x through slit 2

So:

total probability = sum of the two path probabilities



USING QUANTUM RULES TO FIND THE PROBABILITY OF LANDING AT POSITION X

Expanding the Quantum Rule: $|\alpha_1 + \alpha_2|^2$

Step 1: Magnitude definition:

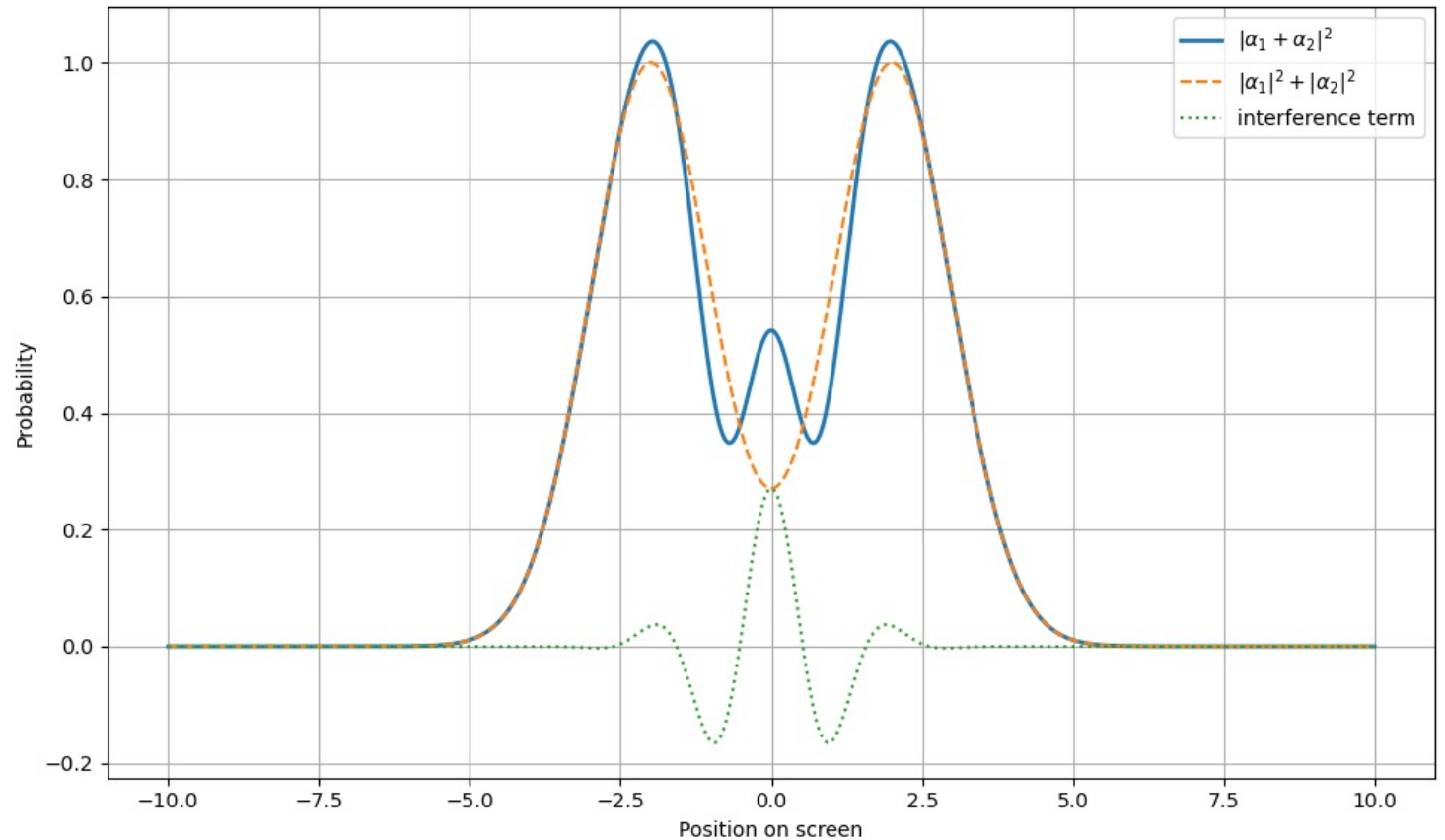
Step 2: Distribute conjugate
 $= (\alpha_1 + \alpha_2)(\overline{\alpha_1} + \overline{\alpha_2})$

Step 3: Multiply out
 $= \alpha_1 \overline{\alpha_1} + \alpha_1 \overline{\alpha_2} + \alpha_1 \overline{\alpha_2} + \alpha_2 \overline{\alpha_2}$

Step 4: Recognise terms
 $= |\alpha_1|^2 + |\alpha_2|^2 + \alpha_1 \overline{\alpha_2} + \alpha_1 \overline{\alpha_2}$

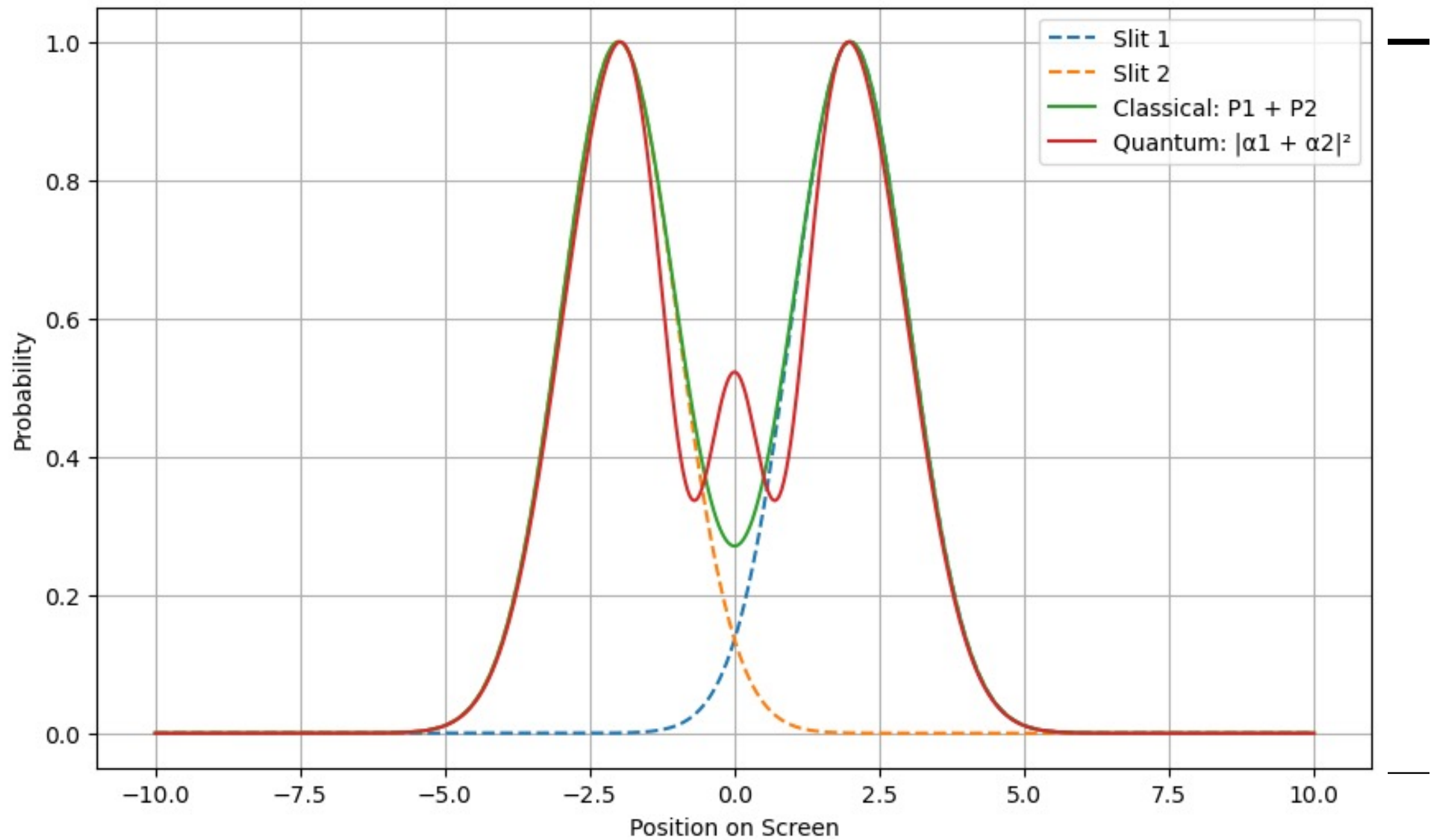
Step 5: Combine conjugate pair
 $\alpha_1 \overline{\alpha_2} + \alpha_1 \overline{\alpha_2} = 2\text{Re}(\alpha_1 \overline{\alpha_2})$

Finally:
 $|\alpha_1 + \alpha_2|^2 = |\alpha_1|^2 + |\alpha_2|^2 + 2\text{Re}(\alpha_1 \overline{\alpha_2})$

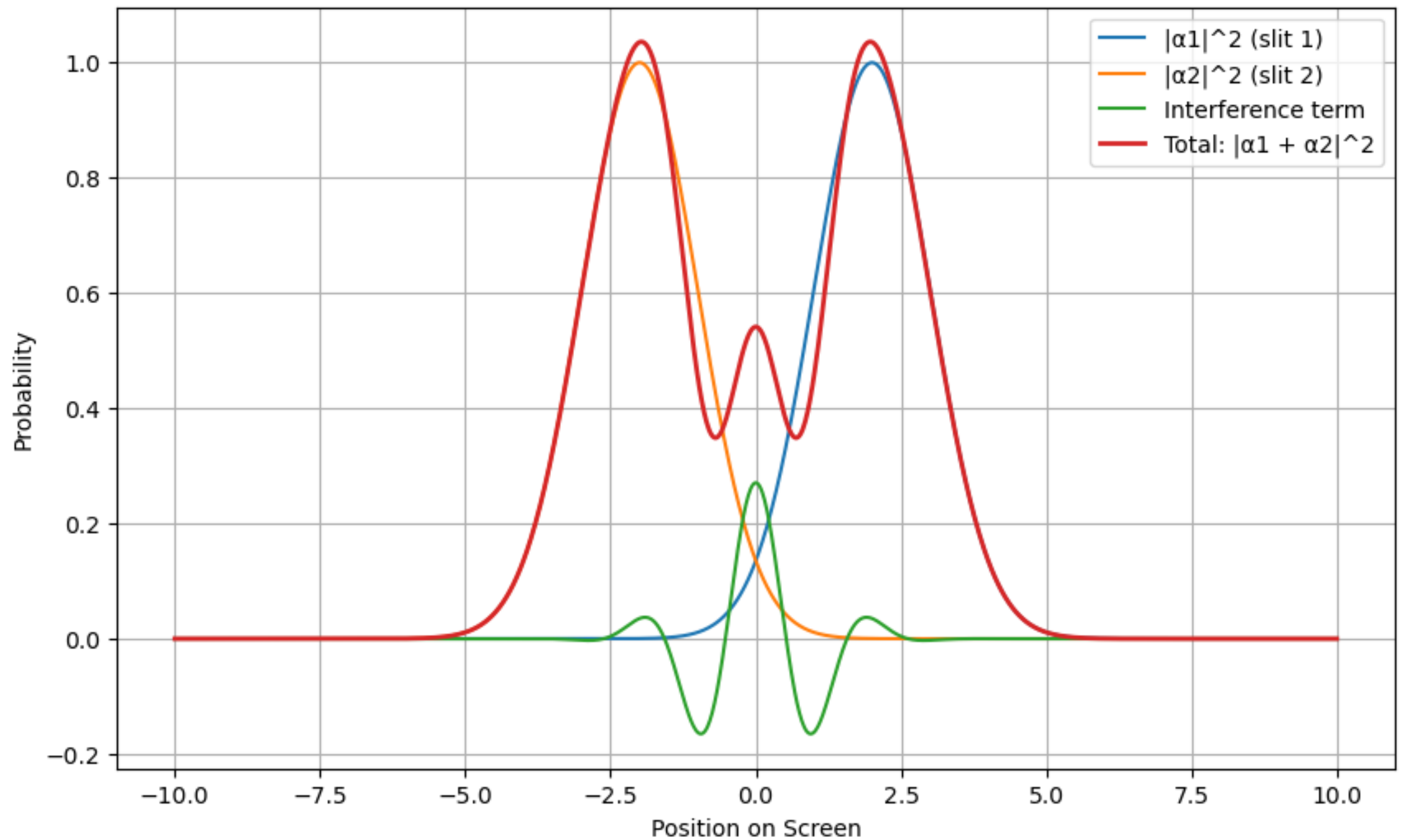


$\text{Re}(z)$ means: **the real part of a complex number z**

Classical vs Quantum Two-Slit Behaviour



Decomposition of Quantum Probability (Two-Slit)



Single-slit pattern



Double-slit pattern



Single-slit pattern



Double-slit pattern



This pattern of alternating light and dark comes from the addition of complex amplitudes

Single-particle interference

Add the experiment, preparation, and detection procedure.

Explain interference between indistinguishable alternatives for a single quantum system.
Avoid describing it as photons needing to collide with one another.

Which-path information and coherence

Compare coherent alternatives with distinguishable paths. Connect the observed distributions to amplitude addition and probability addition.

Distinguish loss of local coherence from conditioning on a measurement outcome.