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The Effects of Bank Size on Equilibrium Interest Rate Dispersion

Mark A. Ehlen
Sandia National Laboratories, Albuquerque, NM 87185

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Abstract

Banks operating in the same local market tend to offer widely different interest rates on essentially the same savings or credit account. This rate dispersion could be attributable to market power or different production costs but, as found by our model, could also be simply due to differences in the number of branches each bank operates. A search model of the local market for deposit and loan accounts is constructed to determine the effect of banks of different sizes on the distribution of interest rates. Consumers with heterogenous search costs shop for the bank with the best interest rate on an account, and heterogeneous banks set prices that maximize profits. The Nash equilibrium is a unique non-degenerate distribution of interest rates.

1 Introduction

Individual markets of goods or services generally exhibit a wide dispersion of prices, even for identical goods.[1] One rational for this dispersion is that consumers shop for goods at some personal cost. When this cost is large, shoppers shop at only a subset of all stores selling the goods and eventually purchase from the one that offers the best price. In this case, the law of one price need not hold: a firm that knows consumers shop at a cost can charge a price that is (1) different from other firms' prices and (2) is above its own unit costs. Given reasonable assumptions about firms' pricing strategies and consumer search costs, there is always some positive probability that among the set of shoppers that visit a given firm, some will receive their best quote from this firm and will see insufficient benefit from additional shopping. (The price reduction expected from shopping at one more store is less than the personal cost of the visit.) The lack of absolute, Bertrand-style price competition allows those with relatively inefficient production technologies to sell at a price higher than the purely competitive, marginal-cost price.

Models of equilibrium price dispersion can be divided into four groups: (1) ones where consumers have the same search costs and firms have the same production

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costs [10, 3, 22, 2], (2) ones where consumers have different search costs but firms have the same production costs [18, 19, 21, 20], (3) ones where consumers have the same search costs but firms have different search costs [15, 9], and (4) ones where both have differing costs [9, 4] (which includes our model). For the monopoly case, Salop[18] showed that while a monopolist facing inelastic demand will charge the (single) monopoly price to all customers, when facing elastic demand he will be "noisy" in his pricing — offering a random distribution of prices he can sort consumers into sub-markets that permit price discrimination. Under certain conditions, the monopolist's dispersed-price profits exceed the single-price profits.

Almost without exception,[2] these models assume that the probability a consumer will visit a particular firm is constant across firms, that is, the market visibility is the same for all firms. This is a significant restriction: for example, a large firm with many stores and extensive advertising should have a much higher probability of being visited than a firm with only a single store and little advertising. This is especially true in the banking sector, where banks attract local customers by operating many branches.

The banking industry exhibits significant dispersion of its "prices," namely the interest rates it quotes for deposit and loan accounts. Both Osborne and Wendel[12] and Neuberger and Zimmerman[11] note significant dispersion of interest rates for the same type of deposit account in the same city. Ehlen[5], using 1987-1994 weekly interest rates on the deposit and loan accounts of the largest banks in eight U.S. cities, found interest rate dispersion to be large and very structured in some cities. For example, in New York City the rank order of interest rates on money market deposit accounts (MMDA) at the five largest banks were strikingly stable. One bank had the highest interest rate most of the time, another bank had the lowest interest rate most of the time, and the rank of the other three were very stable. In addition, the bank with the highest interest rate had the largest number of branches, the largest share of deposits, and the lowest production costs,[3] suggesting some relationship between bank size and rank in the distribution of interest rates. All five banks' weekly fluctuations were closely correlated with changes in the six-month T-bill rate, suggesting that each bank was operating on fixed margins between securities market rates and the rates they offered on deposit and loan accounts.[4]

To examine the effect of bank size on this equilibrium dispersion of bank interest rates, we construct a model of interest rate dispersion where banks of different size and production costs offer (sell) deposit and loan accounts. By size we mean the market "presence" or visibility of the bank to consumers. While there may be correlation between bank deposits and its visibility, the differences can, as shown here, create equilibrium price dispersion. Consumers shop for accounts at different personal costs. Banks, knowing that customers shop at different costs, offer interest

[2] Butters[3] constructs a model with varying probability by allowing firms to choose different levels of "junk mail" advertising.

[3] The production cost figures are quarterly, not weekly, data

[4] The other seven cities exhibited dispersion and rank correlation to lesser degrees, and were found to be correlated with average market rates, the number of banks in the market, and the ratio of thrifts to banks.

rates that maximize their expected profits given the interest rates of rival banks. An equilibrium distribution of interest rates is derived and proved to be unique. The effects of bank market presence, bank production costs, and consumer search costs are seen to affect market rates in ways important to banking policy.

2 The Model

2.1 Consumers

Our one-period economy is composed of M consumers, each with a different but constant marginal cost of search, and N banks that sell D deposit and L loan accounts. We first construct each bank's expected demand for representative deposit account d, then construct each bank's profit maximization based on all of the deposit and loan accounts it offers, and — assuming separateness of all deposit and loan account markets — then find the unique equilibrium distribution of interest rates for deposit account d.

For deposit account d, we assume for clarity that the M consumers have unit inelastic demand and the same minimum-willingness-to-accept interest rate r_{dmin}. Each consumer knows the overall distribution of interest rates — an assumption we substantiate below, but does not know which bank is offering which rate. Shopping for the highest interest rate is costly, so shoppers follow a sequential search rule: first they sample an interest rate from a bank. If the expected increase in offered interest rate from one more sample is greater than the consumer's cost of that search, he will sample from another bank (knowing he can always return to any bank and purchase a deposit account at the interest rate he was quoted). If the expected increase is less than the cost of search, he stops shopping and deposits his money at the bank that quoted the best interest rate.

The banks do not know the search cost of each customer; if they did, then they could offer a rate that makes it not cost effective for the customer to continue to another bank. (This would be *first-degree price discrimination.*) Banks could also tacitly collude and all offer the lowest interest rate acceptable, r_{dmin}. As shown below, however, it is optimal for each bank to offer an interest rate targeted toward a specific subset of customers that come to the bank: (1) whose best interest rate offered thus far is this bank's, and (2) whose expected benefit of an additional search is less than the cost of their search. This bank knows with certainty that these shoppers will purchase their deposit account.

We label each bank according the ordinal rank of the interest rate it offers on account d: Bank 1 offers the lowest interest rate, Bank 2 offers the second-lowest, and so on; therefore, $r_1 < r_2 < r_n < r_N$. (We assert for now that this can be an equilibrium distribution of rates, saving the proof for later.) If a consumer visits bank n and is quoted interest rate r_n, the expected benefit of one additional visit rates that maximize their expected profits given the interest rates of rival banks. An equilibrium distribution of interest rates is derived and proved to be unique. The effects of bank market presence, bank production costs, and consumer search costs are seen to affect market rates in ways important to banking policy.

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$$x_n = \begin{cases} \sum_{h=n+1}^{N} (r_h - r_n)\pi_h, & n < N \\ 0, & n = N \end{cases} \tag{1}$$

where π_h is the probability of visiting bank h and $\sum_{n=1}^{N} \pi_n = 1$. This probability represents the bank's share of market presence or visibility to the consumer. It could be a function of the market share of bank branches, advertising, or of sales (in markets where firms rely on repeat purchases). Consumers with a per-search cost of x will stop shopping for rates when $x_n \leq x$. We represent the distribution of search costs across the M consumers with the continuous cumulative function

$$G(x) = \begin{cases} \frac{x}{sM}, & 0 \leq x \leq sM \\ 1, & x > sM \end{cases} \tag{2}$$

where s is a constant and $dG(x) = g(x)$ is the probability that a (sampled) consumer has a per-search cost of x. It follows that the maximum search cost among consumers is sM. So that a non-zero level of shoppers will purchase from Bank 1, we set $sM > x_1$.

We derive the expected demand for each bank by first calculating it for the bank with the lowest rate r_1 (Bank 1), next calculating for the bank with the second-lowest interest rate r_2, and finally generalizing to Bank n. Consider Bank 1: a consumer with search cost x has a probability π_1 of visiting the bank, and will purchase with probability one if his cost of search x is greater than the expected benefit of an additional search x_1, that is, if $x > x_1 = \sum_{h=2}^{N}(r_h - r_1)\pi_h$. Likewise, he will purchase with probability zero of $x \leq x_1$ (he'll move on). Integrating over the range of search costs, the expected demand for Bank 1 is

$$\begin{aligned} q_1 &= \int_{x_1}^{x^{max}} 1 \times \pi_1 g(x) M dx + \int_0^{x_1} 0 \times \pi_1 g(x) M dx \\ &= \pi_1[G(x^{max}) - G(x_1)]M, \\ \iff \frac{q_1}{M} &= \pi_1[G(x^{max}) - G(x_1)] \end{aligned} \tag{3}$$

Since $[G(x^{max}) - G(x_1)] < 1$, market share $\frac{q_1}{M}$ is strictly less than market presence π_1. Said another way, the number of customers that buy is fewer than the total number that visits. In addition, Bank 1 gets only the fraction π_1 of shoppers whose search costs are so high that they purchase from the first bank they visit.

Now consider Bank 2, which offers the second-lowest rate r_2. Its expected demand comes from two groups of consumers. First, it will get its share, π_2, of the high-search-cost consumers who always purchase at the first bank they visit (those with $x > x_1$). It will also expect demand from consumers with search cost x that satisfies $x_2 < x \leq x_1$; these consumers are either shoppers who arrive at Bank 2

[5] For mathematical tractability we assume that consumers sample with replacement.

on their first visit or had previously visited Bank 1 and their expected benefit encouraged them to move on. Since this second set of consumers will never purchase from Bank 1, the effective probability that Bank 2 has of being visited by these consumers is $\frac{\pi_2}{1-\pi_1}$ (see appendix for derivation). The expected demand for Bank 2 is then the sum of demand from a π_2 fraction of search-once customers plus a $\frac{\pi_2}{1-\pi_1}$ fraction of customers with search costs that satisfy $x_2 < x \leq x_i$:

$$\begin{aligned} q_2 &= \int_{x_1}^{x^{max}} \pi_2 g(x) M dx + \int_{x_2}^{x_1} 1 \times \frac{\pi_2}{1-\pi_1} g(x) M dx \\ &= \pi_2[G(x^{max}) - G(x_1)]M + \left(\frac{\pi_2}{1-\pi_1}\right)[G(x_1) - G(x_2)]M, \qquad (4) \\ \Longleftrightarrow \frac{q_2}{M} &= \pi_2[G(x^{max}) - G(x_1)] + \left(\frac{\pi_2}{1-\pi_1}\right)[G(x_1) - G(x_2)]. \end{aligned}$$

Iterating backwards, the expected demand (expressed as market share) for Bank n charging the nth rank-ordered price is

$$\frac{q_n}{M} = \begin{cases} \pi_n[G(x^{max}) - G(x_1)] + \pi_n \sum_{h=2}^{n} \frac{G(x_{h-1}) - G(x_h)}{1 - \sum_{k=1}^{n=1} \pi_k}, & n > 1 \\ \pi_n[G(x^{max}) - G(x_1)], & n = 1 \end{cases} \qquad (5)$$

Substituting in the distribution of search costs and algebraically manipulating the N equations, the expected demand for the nth bank is[6]

$$\begin{aligned} q_n &= \pi_n[M - \frac{1}{s}(\overline{r} - r_n)], \text{where } \overline{r} = \sum_{n=1}^{N} \pi_n r_n, n = 1, ..., N. \\ \Longleftrightarrow \frac{q_n}{M} &= \pi_n[1 - \frac{1}{sM}(\overline{r} - r_n)] \qquad (6) \end{aligned}$$

Demand is a function of the probability share of Bank n in the market, the distribution of search costs, and the distribution of interest rates. If a bank's rate is above the market average, it will have market share larger than its market presence, and when its rate is below-average, its share will be lower than its market presence. Search costs drive the sensitivity of this swinging around the presence. When search costs are very high, market share approaches the market presence — each shopper only visits once and then purchases. When search costs are relatively low, those with the best rates get inordinately large market shares, and vice versa.

Taking partials of eq.6 shows how banks of certain sizes benefit most from changes in the rates they offer. Denoting $Q = [q_1 \; q_2 \; ... \; q_N]'$ as the demand quantities and $r = [r_1 \; r_2 \; ... \; r_N]'$ as the vector of market prices, we construct the matrix of partials $\frac{dQ}{dr}$:

$$\frac{\partial q_n}{\partial r_d} = \frac{1}{sM} \begin{bmatrix} \pi_1(1-\pi_1) & -\pi_1\pi_2 & \dots & -\pi_1\pi_{N-1} & -\pi_1\pi_N \\ \pi_2\pi_1 & \pi_2(1-\pi_2) & \dots & -\pi_2\pi_{N-1} & -\pi_2\pi_N \\ \dots & \dots & \dots & \dots & \dots \\ \pi_{N-1}\pi_1 & -\pi_{N-1}\pi_2 & \dots & \pi_{N-1}(1-\pi_{N-1}) & -\pi_{N-1}\pi_N \\ \pi_N\pi_1 & -\pi_N\pi_2 & \dots & -\pi_N\pi_{N-1} & \pi_N(1-\pi_N) \end{bmatrix} \qquad (7)$$

[6] See Ehlen[5] for the derivation.

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Demand is a function of the probability share of Bank n in the market, the distribution of search costs, and the distribution of interest rates. If a bank's rate is above the market average, it will have market share larger than its market presence, and when its rate is below-average, its share will be lower than its market presence. Search costs drive the sensitivity of this swinging around the presence. When search costs are very high, market share approaches the market presence — each shopper only visits once and then purchases. When search costs are relatively low, those with the best rates get inordinately large market shares, and vice versa.

Taking partials of eq.6 shows how banks of certain sizes benefit most from changes in the rates they offer. Denoting $Q = [q_1\ q_2\ ...\ q_N]'$ as the demand quantities and $r = [r_1\ r_2\ ...\ r_N]'$ as the vector of market prices, we construct the matrix of partials $\frac{dQ}{dr}$:

$$\frac{\partial q_n}{\partial r_d} = \frac{1}{sM} \begin{bmatrix} \pi_1(1-\pi_1) & -\pi_1\pi_2 & \dots & -\pi_1\pi_{N-1} & -\pi_1\pi_N \\ \pi_2\pi_1 & \pi_2(1-\pi_2) & \dots & -\pi_2\pi_{N-1} & -\pi_2\pi_N \\ \dots & \dots & \dots & \dots & \dots \\ \pi_{N-1}\pi_1 & -\pi_{N-1}\pi_2 & \dots & \pi_{N-1}(1-\pi_{N-1}) & -\pi_{N-1}\pi_N \\ \pi_N\pi_1 & -\pi_N\pi_2 & \dots & -\pi_N\pi_{N-1} & \pi_N(1-\pi_N) \end{bmatrix} \qquad (7)$$

[6]See Ehlen[5] for the derivation.

Row i denotes the partials for Bank i's demand function with respect to a change in column (Bank) j's interest rate. For Bank n, increases in its own rate increase its demand (*ceteris paribus*). A bank with 50 percent of the market presence (and thus the largest) affects its own demand the most through increases in its own interest rate (the value $\pi_n(1-\pi_n)$ is the maxima when $\pi_n = \frac{1}{2}$). Increases in other banks' rates, on the other hand, decrease a bank's expected demand. The two banks with the highest market presence are affected most by a price change by the other (that is, $\pi_i \pi_j$ is maximized). High search costs make the market un-responsive to changes individual bank's prices, while low search costs increase this asymmetric responsiveness of the market to individual price changes.

Before we define the conditions for which there can be an equilibrium price distribution function $F(\cdot)$, where $\hat{r}$ is the solution vector of all rates in the market, we describe the bank production technology that supports the separateness of individual deposit and savings accounts, and define the conjectures that each bank has about its rivals' responses to its pricing strategies.

2.2 Bank Profit Maximization

Before solving for an equilibrium distribution of interest rates for each account, we need to show that each market for a deposit or loan account can be treated separately, i.e., there are no cross-price elasticities between accounts. If a bank increases its savings rate, *ceteris paribus*, the expected demand for loans should not change, and vice versa. If a bank increases interest rate on a short-term loan account increases, the expected demand for another should not change. Separateness allows us to solve the profit maximization problem and prove existence of an equilibrium interest rate, one account at a time.

Separateness is a reasonable assumption for consumers of a particular account since they are rarely affected by the interest rates on the other loan and deposit accounts. Banks remove any arbitrage opportunities that could cause interdependence. For example, if when a loan rate is less than deposit rate, customers could borrow from the bank (buy low) and then deposit it at the same bank (sell high). If a short-term loan rate is higher than a long-term rate, customers could take out a long-term loan (buy low) and lend it at the short-term rate (sell high).

Banks do, though, consider all savings and loan accounts when maximizing expected profits. When pricing their accounts, banks take into consideration their costs of producing these accounts, the opportunity costs of all deposit and loan accounts, and the pricing behavior of the other banks in the market. Our bank production technology is composed of (1) a marginal cost of producing and processing each type of account, c_{nj} (where $j = 1, ..., (D+L)$), and (2) a fixed production cost C_n. Marginal costs are calculated per account dollar instead of per account. Banks take into consideration (1) the opportunity cost of getting loan funds by attracting deposits (vis a vis purchasing them on the wholesale market) and (2) the opportunity cost of using deposits to fund loans (vis a vis investing them in the securities market).

One perspective from which to think about this is the fundamental mechanism of a financial intermediary: to convert short-term liabilities (deposits) into long-term assets (loans). This conversion can be costly on both sides of the balance sheet. For example, when a bank makes a loan, it must not only consider the labor, materials, and other capital necessary to make the loan available — as well as the deposit interest rate and the lost revenue that results from the fraction ρ of deposits that must by law be held in reserve — but it must also consider the return it could receive if it instead invested loan funds in securities. If net returns are greater in the national securities market than the local loan market, a bank may increase profits by diverting funds away from loans. A bank's loan rates — net of production costs and the cost of funds used for these loans (e.g. deposits) — must be greater than securities rates over comparable maturities for it to want to make new loans.

Analogously, when a bank sets its deposit rates, it must take into account the relative cost of funds if they instead purchase them from the wholesale funds market. If in the process of intermediating between short-term deposits and long-term loans a bank has an excess (deficit) of funds, then this net amount can also be sold (bought). In this way the national funds rate plays a key role in determining local market interest rates. We approximate the non-interest costs and interest income or expense for a balance of S_n market securities by the constant marginal cost c_{ns} and market interest rate r_s, respectively.

Finally, a bank must consider how other banks will respond to its pricing decisions. If a bank decreases its loan rate, it might expect its rivals to also decrease their rates; rate increases may be responded to in an analogous way. Formally, Bank n is assumed to have conjectures about $\frac{\partial r_{kj}}{\partial r_{ij}}$, how rival Bank k will change its price when Bank i changes its price. Specifically, we assume that each bank's conjectures are correct, that they match observed behavior by other banks, and that they are consistent with the price elasticities in eq. 7

To summarize, each bank maximizes profits by (1) maximizing returns on loans and funds invested in securities, and (2) minimizing the cost of deposits and other liabilities purchased from the securities market, while (3) taking consumer search behavior, expected demand, and conjectured responses from other banks as given. For Bank n we denote

c_{nd} as the non-interest variable cost of processing account d,

c_{ns} as the non-interest variable cost of processing securities,

S_n the balance of securities,

C_n as fixed production costs,

K_n as total bank capital,

$\hat{\mathbf{r}}_{\mathbf{d}} = [r_{1d}\ r_{2d}\ ...\ r_{Nd}]'$ as the vector of market prices for the dth product,

$\hat{\mathbf{r}}_{\mathbf{n}} = [r_{n1}\ r_{n2}\ ...\ r_{n(D+L)}]'$ as the vector of Bank n's prices, and

$q_{nd}(\hat{\mathbf{r}}_{\mathbf{d}})$ as Bank n's expected demand, as a function of the vector of market rates for account d.

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$\mathbf{\hat{r}_n} = [r_{n1}\ r_{n2}\ \ldots\ r_{n(D+L)}]'$ as the vector of Bank n's prices, and

$q_{nd}(\mathbf{\hat{r}_d})$ as Bank n's expected demand, as a function of the vector of market rates for account d.

Each bank's problem is to choose its own vector of interest rates $\hat{\mathbf{r}}_\mathbf{n}$ that maximizes the profit on loans and securities less the cost of deposits and the fixed cost of production:

$$\begin{aligned} \text{Max } \Pi^i &= \sum_{l=L+1}^{D+L} (r_{nl} - c_{nl}) q_{nl}(\hat{\mathbf{r}}_\mathbf{l}) + (r_s - c_{ns}) S_n \\ &- \sum_{d=1}^{(} D)(r_{nd} + c_{nd}) q_{nd}(\hat{\mathbf{r}}_\mathbf{j}) - C_n, \end{aligned} \tag{8}$$

subject to the balance sheet condition that assets (loans plus securities) equals liabilities (available deposits plus capital):

$$\sum_{l=L+1}^{D+L} q_{nl}(\hat{\mathbf{r}}_\mathbf{j}) + S_n = (1-\rho) \sum_{d=1}^{D+L} q_{nd}(r_{nd}) + K_n \tag{9}$$

Each bank in effect solves $(D+L)$ separate maximizations subject to the consumer demand for the account, the securities rate, production costs and conjectured rival responses. To find the optimal rate r_{nj} for account d, we first solve eq. 9 for S_n and substitute into eq. 8:

$$\begin{aligned} \text{Max } \Pi^n &= \sum_{l=D+1}^{D+L} (r_{nl} - c_{nl}) q_{nl}(\hat{\mathbf{r}}_\mathbf{l}) \\ &+ (r_s - c_{ns}) \left((1-\rho) \sum_{d=1}^{D} q_{nd}(r_{nd}) K_n - \sum_{d=1}^{D+L} q_{nd}(\hat{\mathbf{r}}_\mathbf{d}) \right) \\ &- \sum_{d=1}^{D+L} (r_{nd} + c_{nd}) q_{nd}(\hat{\mathbf{r}}_\mathbf{d}) - C_n, \end{aligned} \tag{10}$$

The first order condition with respect to r_{ij} is necessary and sufficient for the optimal solution under minor assumptions.[7] And as shown below, the conditions allow the equilibrium price dispersion in each market to be independent of the prices in the other account markets.

2.3 Equilibrium Interest Rate Dispersion

We now prove that, given the expected market demand for account d described in eq. 6 and the market supply for account d described by the first order conditions for eq. 11, we can solve for equilibrium price and quantity, where in this case equilibrium price is a distribution of interest rates linked closely to the distribution of market presences. We first define mathematically a *bank equilibrium* and a *market equilibrium* and then show how, under certain conditions, the equilibrium will display a distribution of interest rates that is dependent on the different bank sizes.

[7]To make second order conditions strictly negative, we assume, like Iwata[6], that the conjecture partials $\frac{\partial r_{kj}}{\partial r_{ij}}$ are constant and less than 1. Constancy of partials with respect to interest rate follow from eq. 7. Second order conditions are strictly negative if one of the conjecture is strictly less than zero. See Ehlen[5] for derivations.

As noted by Rothschild[17], equilibrium price dispersion exists only if it is viable and optimal for firms to quote a distribution of prices. We must prove then that given the different sampling probabilities, the distribution of search costs across consumers and an equilibrium number of banks, there exists at least one equilibrium with a non-degenerate distribution of prices, $F(\cdot)$. We prove existence for account d; the other (D+L - 1) account equilibria follow due to separateness of markets.

We first make two definitions and then prove a proposition using three claims. The first claim shows that the lowest offered rate in the market is r_{dmin}, otherwise all banks can decrease their rates simultaneously. The second shows that there cannot be a single-rate equilibrium. The third shows that under certain conditions there is an equilibrium distribution.

> **Definition 1:** Given a set of account d market output quantities $< q_{nd} >_{n=1}^{N}$ and the reservation price r_{dmin}, a *bank equilibrium* is the set of both price distribution $F(\cdot)$ and bank profits $< \Pi^n >_{n=1}^{N}$ such that $\Pi^n \geq \Pi^n(r_{nd})$ for all r_{nd} in the support of $F(\cdot), n = 1, ..., N$.[8]
>
> **Definition 2:** The triple $(F(\cdot), < \Pi^n >_{n=1}^{N}, < q_{nd} >_{n=1}^{N})$ is a *market equilibrium* if and only if for some fixed r_{dmax} and distribution of search costs $G(\cdot)$, (a) $(F(\cdot), < \Pi^n >_{n=1}^{N})$ is a bank equilibrium and (b) expected demand $< q_{nd} >_{n=1}^{N}$ is generated from the cost-minimizing strategies of consumers facing $F(\cdot)$.

Definition 1 assures that no bank can make strictly greater profits if it unilaterally deviates its offered interest rate from the bank equilibrium. Definition 2 is simply a partial equilibrium, where banks are in Nash equilibrium with other banks and in Stackleberg equilibrium with consumers (they take consumers responses as given).

Proposition: Given the distribution of search costs $G(\cdot)$, reservation price r_{dmin} and search cost s that satisfies

$$\sum_{n=2}^{N} \pi_n(r_{nd} - r_{2d}) < sM < (1 - \pi_n)\left(\frac{(r_s - c_{ns}) - (r_{nd} + c_{nd})}{1 - \frac{1}{sM}(\bar{r}_d - r_{nd})}\right) \tag{11}$$

for all n, there exists a unique market equilibrium with a non-degenerate distribution of interest rates. *Proof:* We prove with the following three claims.

Claim 1: If there exists a single-rate market equilibrium, then $r_1 = r_{dmin}$. *Proof:* Suppose not; then either $r_1 > r_{dmin}$ or $r_1 < r_{dmin}$. Let $(F(\cdot), < \Pi^n >_{n=1}^{N})$ and $(F(\cdot), < \Pi^n >_{n=1}^{N}, < q_{ij} >_{i=1}^{N})$ be the bank and market equilibrium when $r_1 > r_{dmin}$. From eq. 6, if banks tacitly collude and decrease interest rates by a small equal amount ϵ, the new equilibrium quantities will be

$$\hat{q}_{nd} = \pi_n\left[M - \left(\frac{1}{s}\sum_{n=1}^{N}\pi_n(r_{nd} + \epsilon) - \frac{1}{s}(r_{nd} + \epsilon)\right)\right]$$

[8] This definition can also necessarily include a non-degenerate price distribution.

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[8] This definition can also necessarily include a non-degenerate price distribution.

$$
\begin{aligned}
&= \pi_n[M - (\frac{1}{s}(\overline{r}_{nd} + \epsilon) - \frac{1}{s}(r_{nd} + \epsilon))] && (12)\\
&= \pi_n[M - \frac{1}{s}(\overline{r}_{nd} - r_d) - \frac{1}{s}\epsilon + \frac{1}{s}\epsilon \\
&= q_{nd}.
\end{aligned}
$$

By decreasing their rates each bank increases profits without losing deposits. Since $(F(\cdot), < \Pi^n >_{n=1}^N)$ is therefore not a bank equilibrium, $(F(\cdot), < \Pi^n >_{n=1}^N, < q_{nd} >_{n=1}^N)$ is not a market equilibrium. Next, the condition $r_1 < r_{dmin}$ cannot hold since bank 1's expected demand is zero when its rate is less than the minimum-willingness-to-accept rate. Thus the counter claim cannot hold.

Claim 2: There can be no single-price market equilibrium. *Proof:* If all banks charge the same rate, this rate must be r_{dmin} from Claim 1. Without loss of generality, consider a two-bank economy with a (supposed) single-price bank equilibrium at $r_1 = r_2 = r_{dmin}$ If Bank 1 increases its rate by a small amount to $(r_1 + \epsilon)$, in profit terms it will more than offset the rate increase with increased market share $\pi_2(1 - \pi_2)M$ (see eq. 7), thus breaking the single-price bank equilibrium. Since the same applies to a bank in an economy of N banks where all banks are offering the minimum-willingness-to-accept rate, the claim holds.

Claim 3: Given a maximum cost of search sM that satisfies

$$
\sum_{n=2}^{N} \pi_n(r_{nd} - r_{2d}) < sM < (1 - \pi_n)\left(\frac{(r_s - c_{ns}) - (r_{nd} + c_{nd})}{1 - \frac{1}{sM}(\overline{r}_d - r_{nd})}\right) \tag{13}
$$

for $n = 1, ..., N$. No bank will deviate its price from the non-degenerate distribution of interest rates. *Proof:* Assuming that banks have correct conjectures about how rival banks will respond to their interest rate, that is, that conjectures are consistent with observed equilibrium behavior,[9] the banks have in essence perfect and common knowledge about each other's actions. Assume that all banks are initially pricing accounts at the market equilibrium distribution described by $< \pi_n, r_{nd} >_{n=1}^N$. Additionally, from eq. 11 let bank n's net marginal cost $[(r_{1d} - c_{1d}) - (r_s - c_{ns})]$ be strictly greater than zero.[10] We can immediately eliminate one possible deviation: Bank 1 will not decrease its rate since it equals r_{dmin} by Claim 1. Now consider all the other possible deviations. Without loss of generality, let Bank n decrease its interest rate from its equilibrium rate by the small amount ϵ. From eq. 6, the new expected demand for Bank n will be

$$
\begin{aligned}
q_{nd} &= \pi_n\left[M - \left(\frac{1}{s}\sum_{n=1}^{N}\pi_n(r_{nd} + \epsilon) - \frac{1}{s}(r_{nd} + \epsilon)\right)\right] \\
&= \pi_n[M - \frac{1}{s}(\overline{r} - r_{nd})] - \pi_n\frac{1}{s}(1 - \pi_n)\epsilon. && (14)
\end{aligned}
$$

[9] Laitner[8], Bresnahan[1], Perry[13], and Kamien and Schwartz[7] argue that conjectures are believable only if they are consistent with observed behavior.

[10] This lower bound on marginal cost defines the equilibrium number of banks in the market: if Bank 1's equilibrium interest rate isn't greater than its net marginal cost, it will stop production and drop out of the market. This exiting continues until the lowest-rate bank's rate is above cost.

Thus Bank n demand decreases by $\pi_n \frac{1}{s}(\pi_n - 1)\epsilon$. From eq. 11, the total change in profit due to a rate increase of ϵ is

$$\begin{aligned} \Delta\Pi^i &= \Delta[((r_{nd} + c_{nd}) - (r_s - c_{ns}))q_{nd}(\hat{r}_{nd})] \\ &= \Delta r_{nd} q_{nd} + [(r_{nd} + c_{nd}) - (r_s - c_{ns})]\Delta q_{nd} \\ &= \epsilon q_{nd} + ((r_{nd} - c_{nd}) - (r_s - r_{ns}))\frac{1}{s}\pi_n(\pi_n - 1)\epsilon. \end{aligned} \tag{15}$$

No bank will deviate from its equilibrium rate if $\Delta\Pi^n < 0$, or $q_{nd} < [(r_{nd}+c_{nd})-(r_s-c_{ns})]\frac{1}{s}\pi_n(\pi_n - 1)\,\forall\, n$. This is true if the cost of search s is sufficiently small. At the same time, the previous condition $x^{max} > x_{1d}$ requires that $s > \frac{1}{M}\sum_{n=2}^{N}\pi_n(r_{nd} - r_{1d})$. Together, the range of sM values that support a non-degenerate equilibrium distribution are

$$\sum_{n=2}^{N}\pi_n(r_{nd} - r_{2d}) < sM < (1 - \pi_n)\left(\frac{(r_s - c_{ns}) - (r_{nd} + c_{nd})}{1 - \frac{1}{sM}(\bar{r}_d - r_{nd})}\right) \tag{16}$$

Thus, given correct rival conjectures and search costs that are low enough, no bank will deviate its price from the equilibrium value. This completes the proof of the proposition.

3 Discussion

In the context of our model, market presence π_n has at least three interesting and somewhat independent effects on the equilibrium distribution of prices offered on the market. First, as illustrated in eq. 6, banks with relatively poor rates but good presence can capture the same market share of accounts as banks with an opposite set of conditions. Good market exposure pays, especially when consumers search a lot. Second, as illustrated in eq. 7, banks with large market presence benefit most both from offering better rates and from the rate worsening by their rivals. If banks have consistent (i.e., correct) conjectures about the pricing strategies of their rivals, then banks with large market presence have a stronger, arguably more accurate influence on the Nash equilibrium outcome. The lower the cost of search, the stronger is the market's response to changes in individual bank's prices.

Third, market presence (and search costs) determines the existence and uniqueness of equilibrium; for particular sets of $< \pi^n >_{n=1}^{N}$. The left-hand side inequality in eq. 16 insures that a positive fraction of shoppers will actually purchase at each bank. The right-hand side inequality is less intuitive but its constraint is clear: if production costs $c_n s$ and $c_n d$ are the same for all banks, the right-hand term is smallest for large π_n (and thus large r_{nd}); if so, the distribution of prices would degenerate to some other form.

Thus far we have avoided the question of whether and at what level there is a strong correlation in the banking sector between market presence — the "availability" of the bank to customers — and market performance — how many deposit and savings account dollars the bank creates. Small banks can offer a superior rate and advertise heavily, thereby garnering a larger than normal share of a particular deposit market. Likewise, large banks can price themselves out of a market by

Thus Bank n demand decreases by $\pi_n \frac{1}{s}(\pi_n - 1)\epsilon$. From eq. 11, the total change in profit due to a rate increase of ϵ is

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Thus far we have avoided the question of whether and at what level there is a strong correlation in the banking sector between market presence — the "availability" of the bank to customers — and market performance — how many deposit and savings account dollars the bank creates. Small banks can offer a superior rate and advertise heavily, thereby garnering a larger than normal share of a particular deposit market. Likewise, large banks can price themselves out of a market by offering inferior rates and not advertising the account. In reality, the parameter π_n represents a composite of mechanisms (e.g., number of branches, advertising) that influence the probability that a shopper will request price information and ultimately make a purchase. In banking, where there are few physical goods to see in stores or advertising, asset levels are an imperfect measure of market presence. Over time, banks that decrease their production costs can pass on these savings to consumers in the form of better interest rates, resulting in larger market share. If, in fact, the level of market presence π_n is determined by the amount of advertising spend (at a cost of $c_n a$), then, all else being equal, banks with lower production costs can spend more on advertising costs, thereby increasing market presence and share.

4 Conclusions

By introducing an explicit measure of how banks make themselves known to potential customers, we better represent the effect of bank "size" on the interest rates experienced by customers. Particularly in banking, there is not necessarily perfect relationship between market presence and market performance; as illustrated by this model, the two can vary considerably. Banks with high production costs but excellent market presence can maintain levels of accounts equal to those with the lowest costs but poor exposure to potential depositors.

Significant imbalances between these two measures can and should have long-term implications for the survival of the bank. Poor production processes should lead to poor rates, sales, profits, and eventually dollars to expand branches and advertising — two likely mechanisms for maintaining or improving market presence. If market presence goes down, so do sales and eventually viability as a bank. The study of the stability of this equilibrium price dispersion and its dynamics is reserved for future study.

The author would like to thank Bob Avery, Robert Masson, and Bruce Smith for insightful comments and discussion.

5 Appendix

To show that the probability of a consumer with search cost that satisfies $x_{2j} < x < x_{1j}$ will visit and make a purchase from bank 2 equals $\frac{\pi_2}{1-\pi_1}$, we derive this probability for a three-bank economy in which each bank has the same probability π_n of being sampled, and then computing (without loss of generality) for an economy of four banks with different probabilities.

Consider a three-bank version of our economy where $\pi_1 = \pi_2 = \pi_3 = 1/3$. Bank 2 ($B_2$) will target customers with specific search costs; call this customer A_2. Let $t = 1, 2, ..., T$ denote the time intervals over which A_2 shops. The first time A_2 shops ($t = 1$), the probability that he will visit B_2 is 1/3. At $t = 2$, A_2 will still be shopping only if at $t = 1$ he visited B_1 (the bank with the lowest interest rate). Assuming that A_2 samples with replacement, the probability that A_2 visits B_2 at $t = 2$ is

$$\Pr\{A_2 \text{ visits } B_1 \text{ at } t_1\} \times \Pr\{A_2 \text{ visits } B_1 \text{ at } t_1\} = \frac{1}{3}\frac{1}{3} = \left(\frac{1}{3}\right)^2. \tag{17}$$

Similarly, the probability that A_2 visits B_2 at t_3 is

$$\begin{aligned}\Pr\{A_2 \text{ visits } B_1 \text{ at } t_1\} &\times \Pr\{A_2 \text{ visits } B_1 \text{ at } t_1\} \\ &\times \Pr\{A_2 \text{ visits } B_1 \text{ at } t_1\} = \frac{1}{3}\frac{1}{3}\frac{1}{3} = \left(\frac{1}{3}\right)^3.\end{aligned}$$

The total probability that A_2 will visit B_2 in any one of n visits *and* sample a new low price, as n approaches ∞, is

$$\sum_{i=1}^{\infty}(\frac{1}{3})^i = \sum_{i=0}^{\infty}(\frac{1}{3})^i - 1 = \frac{1}{1-\frac{1}{3}} - 1 = \frac{3}{2} - 1 = \frac{1}{2}. \tag{18}$$

which is equivalent to the $\frac{\pi_2}{1-\pi_1}$ term in eq. 5.

Now consider the case of four banks, each with different sampling probabilities. The probability that A_3 visits B_3 at t_1 is π_3. For A_3 to visit B_3 at t_2, he must have sampled B_1 at t_1; thus the total probability that A_3 will visit B_3 at t_2 is

$$\begin{aligned}\Pr\{A_3 \text{ visits } B_1 \text{ at } t_1\} &\times \Pr\{A_3 \text{ visits } B_3 \text{ at } t_2\} \\ &+ \Pr\{A_3 \text{ visits } B_2 \text{ at } t_1\} \times \Pr\{A_3 \text{ visits } B_2 \text{ at } t_2\} \\ &= \pi_1\pi_3 + \pi_2\pi_3 = (\pi_1 + \pi_2)\pi_3.\end{aligned}$$

Similar to the three-bank case, it follows that the probability that A_3 will visit B_3 in any one of n visits (*and* that r_3 will be the best quote at that visit), as n approaches ∞, is

$$\pi_3\sum_{i=0}^{\infty}(\pi_1 + \pi_2)^i = \frac{\pi_3}{1-(\pi_1+\pi_2)} \tag{19}$$

which is equivalent to the $\frac{\pi_n}{1-\sum_{k=h+1}^{N}\pi_k}$ term in eq. 5.

In summary, while π_{N-1} is the probability that *any* shopper will visit bank $N-1$, $\frac{\pi_{N-1}}{1-\pi_N}$ is the probability that an agent with search costs that satisfy $x_{N-1} < x \leq x_N$ will stop shopping and make a purchase.

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